

# Analysis of T-beam Bridge Using Finite Element Method

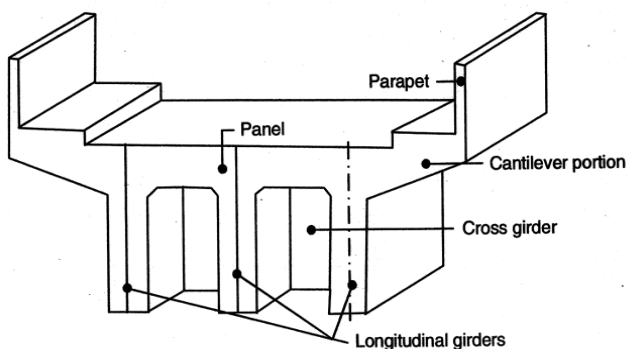
R.Shreedhar<sup>\*</sup> Spurti Mamadapur

**Abstract**— *T-beam bridge decks are one of the principal types of cast-in place concrete decks. T-beam bridge decks consist of a concrete slab integral with girders. The finite element method is a general method of structural analysis in which the solution of a problem in continuum mechanics is approximated by the analysis of an assemblage of finite elements which are interconnected at a finite number of nodal points and represent the solution domain of the problem. A simple span T-beam bridge was analyzed by using I.R.C. loadings as a one dimensional structure. The same T-beam bridge is analysed as a three- dimensional structure using finite element plate for the deck slab and beam elements for the main beam using software STAAD ProV8i. Both models are subjected to I.R.C. Loadings to produce maximum bending moment. The results obtained from the finite element model are lesser than the results obtained from one dimensional analysis, which means that the results obtained from manual calculations subjected to IRC loadings are conservative.*

**Index Terms**—T-Beam, Finite Element Method, IRC Loadings, Courbon's Method.

## I. INTRODUCTION

T-beam, used in construction, is a load-bearing structure of reinforced concrete, wood or metal, with a t-shaped cross section. The top of the T-shaped cross section serves as a flange or compression member in resisting compressive stresses. The web of the beam below the compression flange serves to resist shear stress and to provide greater separation for the coupled forces of bending.



**Fig 1: Components of T-Beam Bridge**

A beam and slab bridge or T- beam bridge is constructed when the span is between 10 -25 m. The bridge deck essentially consists of a concrete slab monolithically cast over longitudinal girders so that the T-beam effect prevails. To impart transverse stiffness to the deck, cross girders or diaphragms are provided at regular intervals. The number of longitudinal girders depends on the width of the road. Three girders are normally provided for a two lane road bridge. T-beam bridges are composed of deck slab 20 to 25cm thick

and longitudinal girders spaced from 1.9 to 2.5m and cross beams are provided at 4 to 5m interval.

## II. BRIDGE LOADING

### A. Dead and Superimposed Dead Load

For general and building structures, dead or permanent loading is the gravity loading due to the structure and other items permanently attached to it. It is simply calculated as the product of volume and material density. Superimposed dead load is the gravity load of non-structural parts of the bridge. Such items are long term but might be changed during the lifetime of the structure. An example of superimposed dead load is the weight of the parapet. There is clearly always going to be a parapet so it is a permanent source of loading. However, it is probable in many cases that the parapet will need to be replaced during the life of the bridge and the new parapet could easily be heavier than the original one. Because of such uncertainty, superimposed dead load tends to be assigned higher factors of safety than dead load.

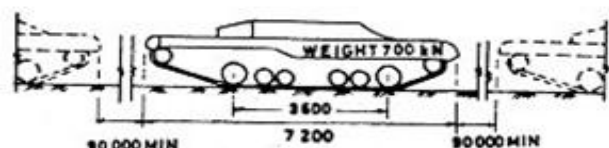
The most notable item of superimposed dead load is the road pavement or surfacing. It is not unusual for road pavements to get progressively thicker over a number of years as each new surfacing is simply laid on top of the one before it. Thus, such superimposed dead loading is particularly prone to increases during the bridge lifetime. For this reason, a particularly high load factor is applied to road pavement. Bridges are unusual among structures in that a high proportion of the total loading is attributable to dead and superimposed dead load. This is particularly true of long-span bridges.

### B. Live loads

Road bridge decks have to be designed to withstand the live loads specified by Indian Roads Congress (I.R.C: 6-2000 sec2)

#### 1. Highway bridges:

In India, highway bridges are designed in accordance with IRC bridge code. IRC: 6 - 1966 – Section II gives the specifications for the various loads and stresses to be considered in bridge design. There are three types of standard loadings for which the bridges are designed namely, IRC class AA loading, IRC class a loading and IRC class B loading



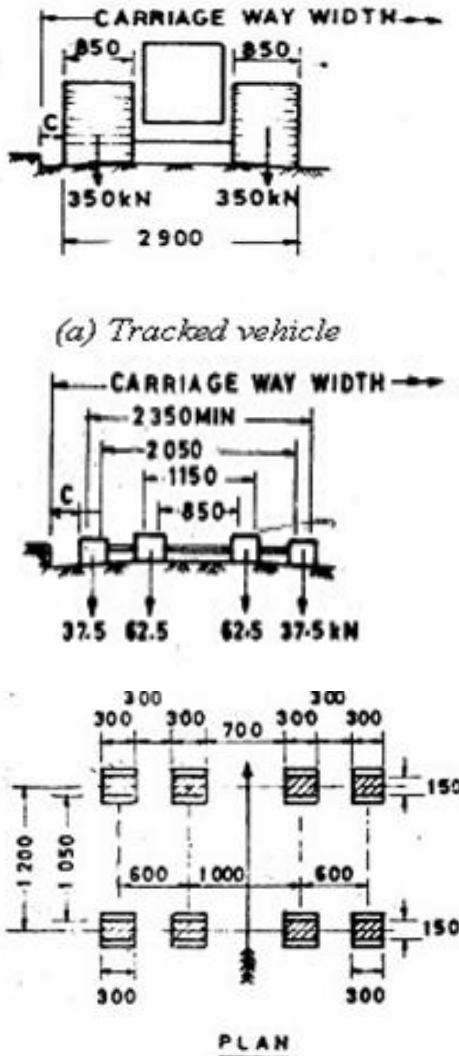


Fig. 2 IRC AA loading

IRC class AA loading consists of either a tracked vehicle of 70 tonnes or a wheeled vehicle of 40 tonnes with dimensions as shown in Fig.2. The units in the figure are mm for length and tonnes for load. Normally, bridges on national highways and state highways are designed for these loadings. Bridges designed for class AA should be checked for IRC class A loading also, since under certain conditions, larger stresses may be obtained under class A loading. Sometimes class 70 R loading given in the Appendix - I of IRC: 6 - 1966 - Section II can be used for IRC class AA loading. Class 70R loading is not discussed further.

Class A loading consists of a wheel load train composed of a driving vehicle and two trailers of specified axle spacing's (FIG 3). This loading is normally adopted on all roads on which permanent bridges are constructed. Class B loading is adopted for temporary structures and for bridges in specified areas. For class A and class B loadings, reader is referred to IRC: 6 -2000 – Section II.

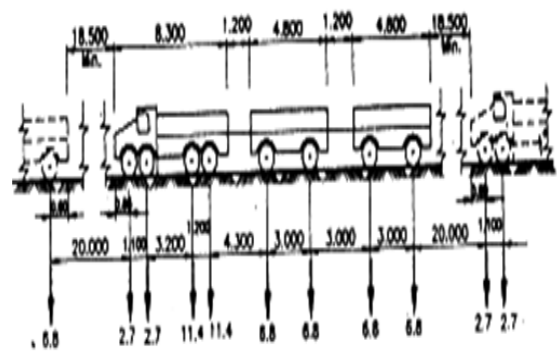


Fig 3 IRC Class a loading

### C. Impact load

The impact factors to be considered for different classes of I.R.C. loading as follows:

a) For I.R.C. class A loading

The impact allowance is expressed as a fraction of the applied live load and is computed by the expression,

$$I = \frac{A}{B+L}$$

Where, I=impact factor fraction

A=constant having a value of 4.5 for a reinforced concrete bridges and 9.0 for steel bridges.

B=constant having a value of 6.0 for a reinforced concrete bridges and 13.5 for steel bridges.

L=span in meters.

For span less than 3 meters, the impact factor is 0.5 for a reinforced concrete bridges and 0.545 for steel bridges. When the span exceeds 45 meters, the impact factor is 0.088 for a reinforced concrete bridges and 0.154 for steel bridges.

b) For I.R.C. Class AA or 70R loading

(i) For span less than 9 meters

➤ For tracked vehicle- 25% for a span up to 5m linearly reduced to a 10% for a span of 9m.

➤ For wheeled vehicles-25%

(ii) For span of 9 m or more

➤ For tracked vehicle- for R.C. bridges, 10% up to a span of 40m. For steel bridges, 10% for all spans.

➤ For wheeled vehicles- for R.C. bridges, 25% up to a span of 12m. For steel bridges, 25% for span up to 23 meters.

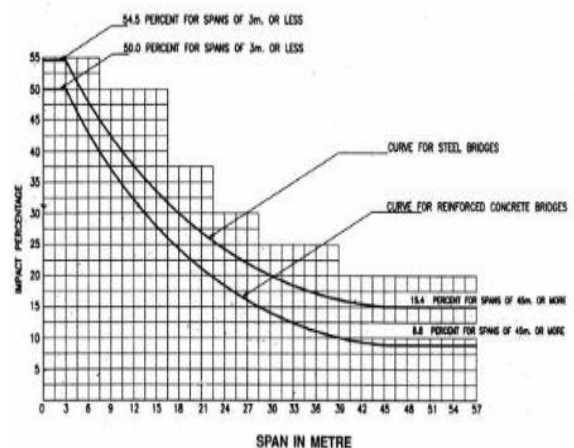


Fig 4: Impact percentage curve for highway bridges for IRC class A and IRC Class B loading

### III. GENERAL FEATURES

A typical tee beam deck slab generally comprises the longitudinal girder, continuous deck slab between the tee beams and cross girders to provide lateral rigidity to the bridge deck.

It is known that the bridge loads are transmitted from the deck to the superstructure and then to the supporting substructure elements. It is rather difficult to imagine how these loads get transferred. If a vehicle is moving on the top of a particular beam, it is reasonable to say that, this particular beam is resisting the vehicle or truckload. However, this beam is not alone; it is connected to adjacent members through the slab and cross girders. This connectivity allows different members to work together in resisting loads. The supporting girders share the live load in varying proportions depending on the flexural stiffness of the deck and the position of the live load on the deck.

The distribution of live load among the longitudinal girders can be estimated by any of the following rational methods.

1. Courbon's method
2. Guyon Massonet method
3. Hendry Jaegar method

#### A. Courbon's Method

Among the above mentioned methods, Courbon's method is the simplest and is applicable when the following conditions are satisfied:

- The ratio of span to width of deck is greater than 2 but less than 4
- The longitudinal girders are interconnected by at least five symmetrically spaced cross girders.
- The cross girder extends to a depth of at least 0.75 times the depth of the longitudinal girders.

Courbon's method is popular due to the simplicity of computations as detailed below:

When the live loads are positioned nearer to the kerb as shown below.

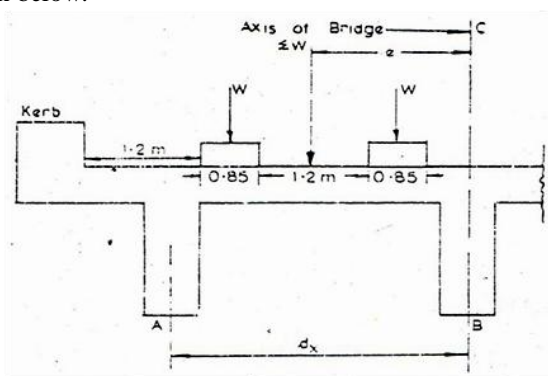


Fig 5: Position of live loads

The centre of gravity of live load acts eccentrically with the centre of gravity of the girder system. Due to this eccentricity, the loads shared by each girder are increased or decreased depending upon the position of the girders. This is calculated by Courbon's theory by a reaction factor given by

$$R_x = \left( \frac{\sum W}{n} \right) \left[ 1 + \frac{\sum I}{\sum d_x^2 I} d_x \cdot e \right]$$

Where,

$R_x$  = Reaction factor for the girder under consideration

$I$  = Moment of inertia of each longitudinal girder

$d_x$  = Distance of the girder under consideration from the central axis of the bridge

$W$  = Total concentrated live load

$n$  = Number of longitudinal girders

$e$  = Eccentricity of live load with respect to the axis of the bridge.

The live load bending moments and shear forces are computed for each of the girders. The maximum design moments and shear forces are obtained by adding the live load and dead load bending moments. The reinforcement in the main longitudinal girders are designed for the maximum moments and shears developed in the girders.

An approximate method may be used for the computation of the bending moments and shear forces in the cross girders. The cross girders are assumed to be equally shared by the cross girders. This assumption will simplify the computation of bending moments and shear forces in the cross girders.

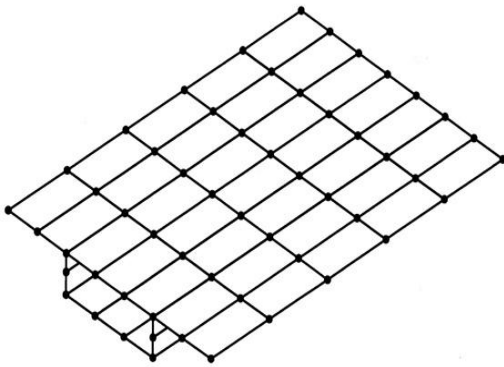
#### B. Finite Element Method

The finite element method is a well known tool for the solution of complicated structural engineering problems, as it is capable of accommodating many complexities in the solution. In this method, the actual continuum is replaced by an equivalent idealized structure composed of discrete elements, referred to as finite elements, connected together at a number of nodes. Thus the finite element method may be seen to be very general in application and it is sometimes the only valid form of analysis for difficult deck problems. The finite element method is a numerical method with powerful technique for solution of complicated structural engineering problems. It is mostly accurately predicted the bridge behavior under the truck axle loading.

The finite element method involves subdividing the actual structure into a suitable number of sub-regions that are called finite elements. These elements can be in the form of line elements, two dimensional elements and three- dimensional elements to represent the structure. The intersection between the elements is called nodal points in one dimensional problem where in two and three-dimensional problems are called nodal lines and nodal planes respectively. At the nodes, degrees of freedom (which are usually in the form of the nodal displacement and or their derivatives, stresses, or combinations of these) are assigned. Models which use displacements are called displacement models and models based on stresses are called force or equilibrium models, while those based on combinations of both displacements and stresses are called mixed models or hybrid models. Displacements are the most commonly used nodal variables, with most general purpose programs limiting their nodal degree of freedom to just displacements. A number of displacement functions such as polynomials and trigonometric series can be assumed, especially polynomials because of the ease and simplification they provide in the finite element formulation. This method needs more time and efforts in modeling than the grillage. The results obtained



from the finite element method depend on the mesh size but by using optimization of the mesh the results of this method are considered more accurate than grillage. Fig. 6 below shows the finite element mesh for the deck slab and also for three-dimensional model of bridge.



**Fig 6: Three- Dimensional Structures Composed of Finite Plate Elements**

### 1 Advantages of Finite element Method

The finite element method has a number of advantages; they include the ability to :

- Model irregularly shaped bodies and composed of several different materials.
- Handle general load condition and unlimited numbers and kinds of boundary conditions.
- Include dynamic effects.
- Handle nonlinear behaviour existing with large deformation and non- linear materials.

### 2 Disadvantages of Finite Element Method

- Commercial software packages the required hardware, which have substantial price reduction, still require significant investment
- FEM obtains only approximate solutions.
- Stress values may vary by 25% from fine mesh analysis to average mesh analysis.
- Mistakes by the user can be fatal.
- It takes longer time for execution.

### 3 Element size and aspect ratio

The accuracy of the results of a finite element model increases as the element size decreases. The required size of elements is smaller at areas where high loads exist such as location of applied concentrated loads and reactions. For a deck slab, the dividing the width between the girders to five or more girders typically yields accurate results. The aspect ratio of the element (length-to-width ratio for plate and shell elements and longest-to-shortest side length ratio for solid elements) and the corner angles should be kept within the values recommended by the developer of the computer program. Typically aspect ratio less than 2 and corner angles between 60 and 120 degrees are considered acceptable. In case the developer recommendations are not followed, the inaccurate results are usually limited to the non conformant elements and the surrounding areas. When many of the elements do not conform to the developer recommendation, it is recommended that a finer model be developed and the results of the two models compared. If the difference is within the acceptable limits for design, the coarser model

may be used. If the difference is not acceptable, a third, finer model should be developed and the results are then compared to the previous model. This process should be repeated until the difference between the results of the last two models is within the acceptable limits. For deck slabs with constant thickness, the results are not very sensitive to element size and aspect ratio. In this study the finite element model was carried out by using STAADPRO 2008

## IV. DESIGN EXAMPLE

Clear width of roadway= 7.5m Span (centre to centre of bearings) =16m

Average thickness of wearing coat = 80mm

Cross section of Deck:

Three main girders are provided at 2.5m centers.

Thickness of deck slab=200mm

Wearing coat=80mm

Width of main girders=300mm

Kerbs 600mm wide by 300mm deep are provided.

Cross girders are provided at every 4m interval.

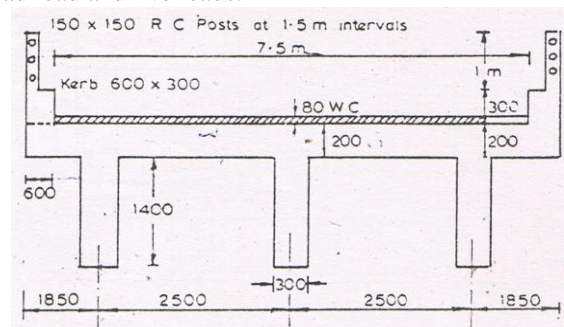
Breadth of cross girder= 300mm.

Depth of main girder= 160cm at the rate of 10cm per meter of span.

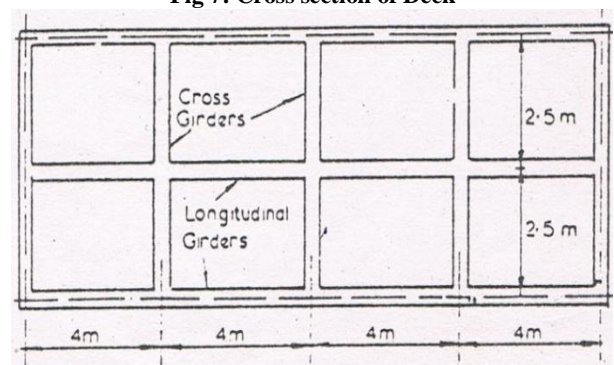
The depth of cross girder is taken as equal to the depth of main girder to simplify the computation.

The bridge is analyzed as follows:

The bridge is first analyzed using I.R.C. specifications. The beam is considered as one dimensional element subject to dead load and live loads.



**Fig 7: Cross section of Deck**



**Fig 8: PLAN OF BRIDGE DECK**

The dead loads include self weight of the structure and 80mm wearing coat.

Live load bending moment:

Case 1: I.R.C. class AA tracked vehicle

Impact factor (for class -AA loads) = 10%

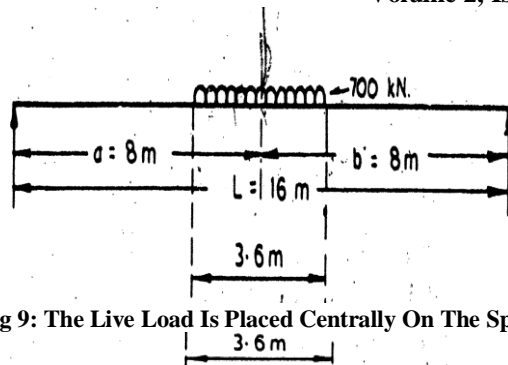


Fig 9: The Live Load Is Placed Centrally On The Span.

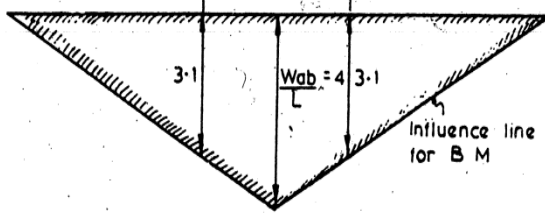


Figure 10: Influence Line for Bending Moment in Girder

Bending moment =  $0.5(4+3.1)700 = 2485 \text{ kN-m}$

Bending moment including impact factor and reaction factor for outer girder

$$= (2485 \times 1.1 \times 0.5536) = 1513 \text{ kN-m}$$

For inner girder

$$= (2485 \times 1.1 \times 0.3333) = 912 \text{ kN-m}$$

Case 2: For two trains of I.R.C. class A loading using Courbon's method.

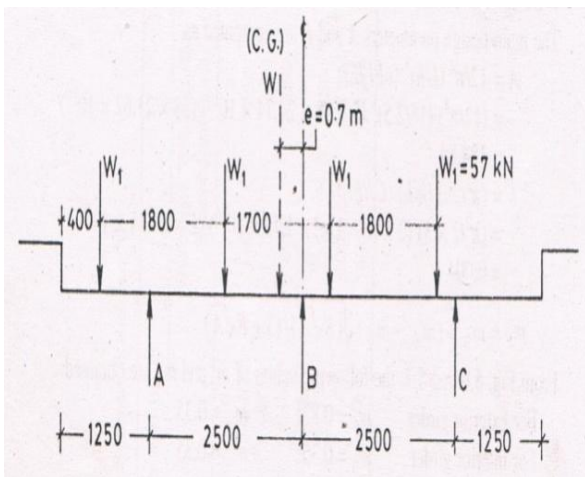


Fig 11: The Cross Section of the Tee Beam and Slab Deck with Two Trains of I.R.C. Class a Loading Positioned to Achieve Maximum Eccentricity of the Centroid of the Loading System

In this case,  $\Sigma W = 4W$ ,  $n = 3$ ,  $e = 0.7 \text{ m}$

Second moment of area is same for all girders.

Reaction Factors (Courbon's Method)

$$R_x = \frac{\Sigma W}{n} \left[ 1 + \frac{e}{\Sigma (d_x^2 + n)} (d_x e) \right]$$

$$R_A = \frac{4W}{3} \left[ 1 + \frac{(31 \times 2.5 \times 0.7)}{(21 \times 2.5^2)} \right]$$

$$R_A = 1.893W$$

$$R_B = \frac{4W}{3} [1 + 0] = 1.333W$$

$$R_C = \frac{4W}{3} [1 - 0.42] = 0.77W$$

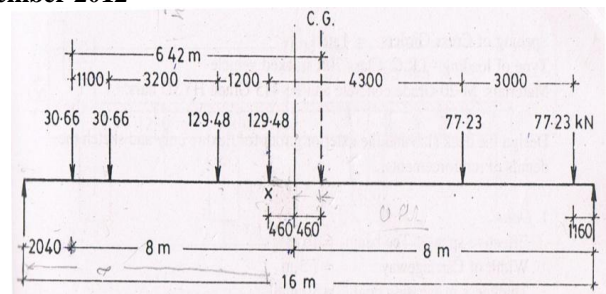


Fig 12: The wheel loads of I.R.C. Class a loading is arranged on the exterior girder A such that the heavier wheel load and the centre of gravity of the load system are equidistant from the centre of the girder.

The centre of gravity of the load system is at a distance of 6.42m from leading wheel.

Maximum bending moment occurs under the 4<sup>th</sup> wheel load from left.

Wheel load = (Train load) \* (Reaction factor) \* (Impact factor)

$$\text{Impact factor} = \left[ \frac{A}{B+L} \right] = \left[ \frac{4.5}{6+16} \right] = 0.20$$

Maximum bending moment X (under 4<sup>th</sup> wheel load from left) is computed as,

$$M_{\max} = (223.72 \times 7.54) - 30.66(5.50 + 4.40) - (129.48 \times 1.20) = 1228 \text{ kN-m}$$

Table 1: Comparison of Live Load and Dead Load Bending Moments

Sl.no	I.R.C loadings	Live load bending moment	Dead bending moment
1	Class AA loading	1513 kN-m	1218 kN-m
2	Class A loading	1228 kN-m	1218 kN-m

## V. MODELLING OF TEE BEAM BRIDGE GIRDER BY FINITE ELEMENT ANALYSIS

Using Staad Pro 2008 software

Span (centre to centre) = 16m

Clear width of road way = 7.5m

Three main girders provided at 2.5m centre

Cross girders provided at 4m interval.

Plate thickness = 200mm

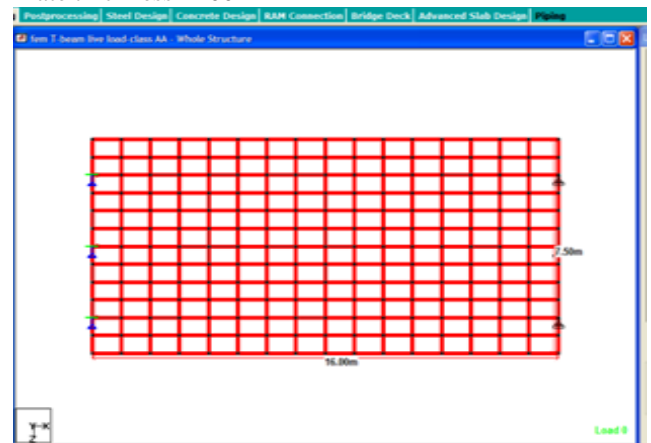
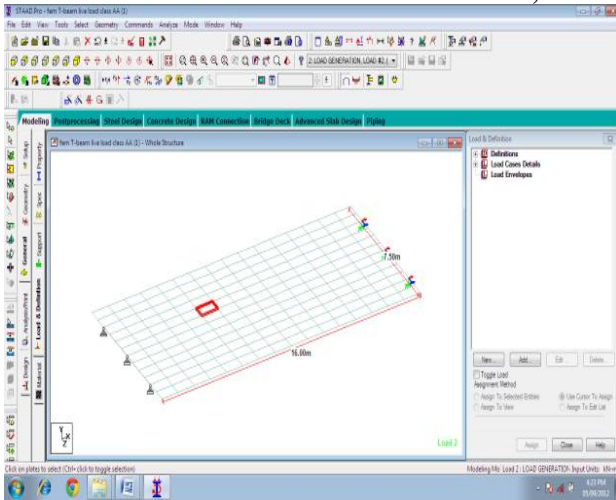
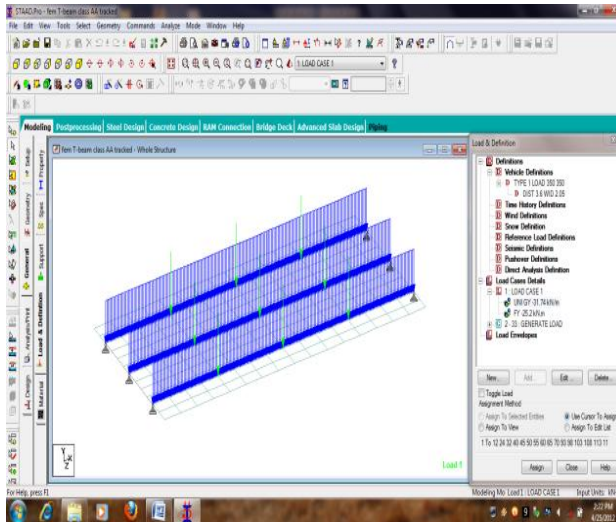


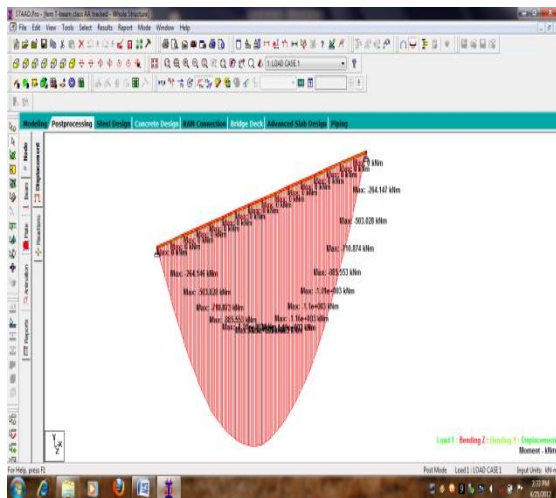
Fig 13: Plate Consisting Mesh of Finite Elements



**Fig 14: Single Element of FEM Plate Model of Aspect Ratio 1.5**



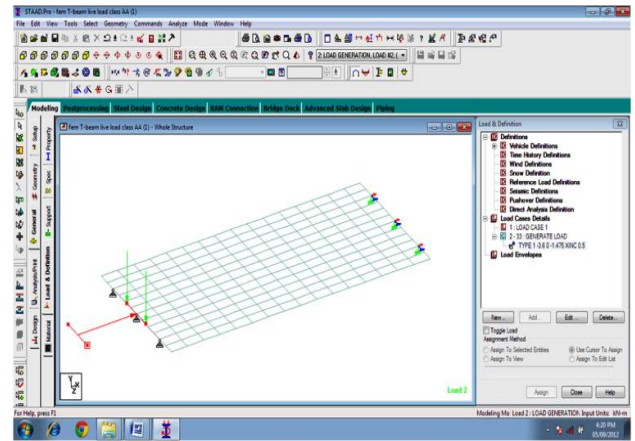
**Fig 15: Dead Load Acting On the Longitudinal Girders (Of UDL Of 31.74 Kn/M and Nodal Load Of 25.2kn/M)**



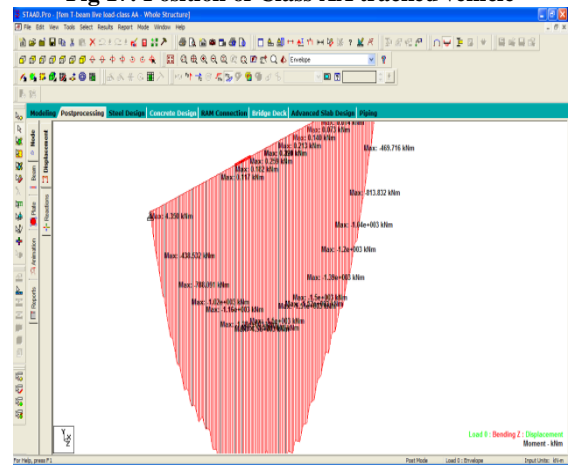
**Fig 16: Maximum Dead Load Bending Moment=1190 Kn-M**

Live load:

CASE 1: Class AA tracked loading

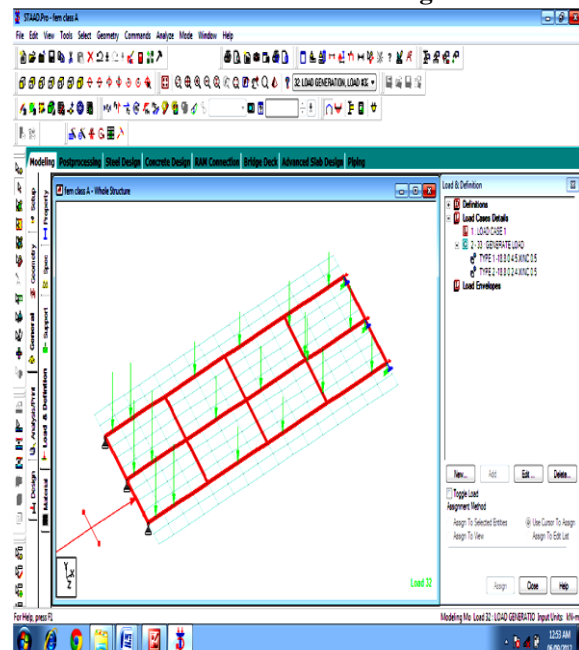


**Fig 17: Position of Class AA tracked vehicle**



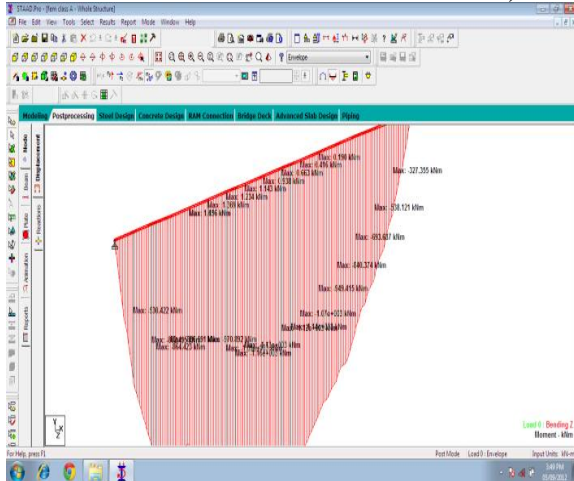
**Fig 18: Maximum live load bending moment for Class AA tracked= 1520kN-m**

Case 2: Class A loading



**Fig 19: Position of live load for Class A**





**Fig 20: Maximum Bending Moment for Live Load Class A=1160 Kn-m**

## VI. RESULTS

**Table 2: Results of Class AA tracked loading**

Sl.no	Class AA	Hand calculation	FEM
1	Dead load B.M	1200kN-m	1190kN-m
2	Live load B.M	1510kN-m	1520kN-m

**Table 3: Results of Class a loading**

Sl.no	Class A	Hand calculation	FEM
1	Dead load B.M	1200kN-m	1190kN-m
2	Live load B.M	1228kN-m	1160kN-m

## VII. CONCLUSION

A simple span T-beam bridge was analyzed by using I.R.C. specifications and Loading (dead load and live load) as a one dimensional structure. Finite Element analysis of a three- dimensional structure was carried out using Staad Pro software. Both models were subjected to I.R.C. Loadings to produce maximum bending moment. The results were analyzed and it was found that the results obtained from the finite element model are lesser than the results obtained from one dimensional analysis, which means that the results obtained from I.R.C. loadings are conservative and FEM gives economical design.

## REFERENCES

- [1] Bridge Design using the STAAD.Pro/Beava", IEG Group, Bentley Systems, Bentley Systems Inc., March 2008.
- [2] "Bridge Deck Analysis" by Eugene J O'Brien and Damien and L Keogh, E&FN Spon, London.
- [3] "Bridge Deck Behavior" by Edmund Hambly, Second Edition, Chapman & hal India, Madras.
- [4] "Design of Bridges" by Krishnaraju, Third Edition, Oxford and IBH Publishing Co. Pvt. Ltd., New Delhi.

- [5] IRC 5-1998, "Standard Specifications And Code Of Practice For Road Bridges" Section I, General Features of Design, The Indian Roads Congress, New Delhi, India, 1998.
- [6] IRC 6-2000, "Standard Specifications and Code of Practice for Road Bridges", Section II, loads and stresses, The Indian Roads Congress, New Delhi, India, 2000.
- [7] "Bridge Design using the STAAD.Pro/Beava", IEG Group, Bentley Systems, Bentley Systems Inc., March 2008.

## ACKNOWLEDGMENT

The authors thank the Principal and Management of KLS Gogte Institute of Engineering, Belgaum for the continued support and cooperation in carrying out this research study.

## AUTHOR BIOGRAPHY



R. Shreedhar is currently Associate Professor in Civil Engineering Department at Gogte Institute of Technology, Belgaum, and Karnataka, India. He is post graduate from National Institute of Technology, Surathkal and pursuing his research in bridges under the guidance of Dr. Vinod Hosur, GIT, and Belgaum.



Spurti Mamadapur is pursuing her Masters degree in Structural Engineering in Civil Engineering Department at Gogte Institute of Technology, Belgaum, and Karnataka, India. She is presently carrying out her dissertation work under the guidance of R. Shreedhar